Chapter 2 Review
Functions: Understanding Rates of Change

<table>
<thead>
<tr>
<th>Average Rate of Change</th>
<th>Instantaneous Rate of Change</th>
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<tbody>
<tr>
<td>• Graphically, represents the slope of the secant over the interval (x_1 \leq x \leq x_2)</td>
<td>• Graphically, represents the slope of the tangent at the point when (x = a)</td>
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<tr>
<td>(\text{avg}_{roc} = \frac{f(x_2) - f(x_1)}{x_2 - x_1})</td>
<td>(\text{inst}_{roc} = \frac{f(a + h) - f(a)}{h}, \ h = \pm 0.01)</td>
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<tr>
<td>• As the interval on either side of (x = a) gets really small, the slope of the secant approaches the slope of the tangent</td>
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Three methods of calculating the instantaneous rate of change:

1. Centred Interval
   - To find the IROC at \(x = 10\), examine the interval \(x \in [9, 11]\)

2. Preceding & Following Intervals
   - Average two intervals nearby: \(x \in [9, 10]\) and \(x \in [10, 11]\)

3. Difference Quotient (most commonly used) \(a = 10\)
   - \(\text{inst}_{roc} = \frac{f(a + h) - f(a)}{h}, \ h = \pm 0.01\)

Speed – Time Graphs

- \(t \in (0, 3)\): speed increases at constant rate \((m > 0)\)
- \(t \in (3, 5)\): speed constant \((m = 0)\)
- \(t \in (5, 6)\): speed decreases at constant rate \((m < 0)\)
At a maximum point, the slopes of the tangents must change from positive to negative.

\[
\text{maximum } m = 0
\]

positive \( m > 0 \)  \hspace{1cm} \text{negative } m < 0

At a minimum point, the slopes of the tangents must change from negative to positive.

negative \( m < 0 \)  \hspace{1cm} \text{positive } m > 0

\[
\text{minimum } m = 0
\]

Example #1: Estimate the IROC of the area of the circle when the radius is 4 cm.

Example #2: For the function \( f(x) = x^2 - 4x \) and the point \( P(2, -4) \), determine:

(a) Avg. ROC for \( 1.5 \leq x \leq 2.5 \)
(b) IROC at point \( P \)
(c) whether the point \( P \) is a maximum or a minimum