2.5 Solving Problems Involving Rates of Change

Recall:
- **Instantaneous rate of change** = slope of tangent
- **Tangent Line**: line which only touches the curve at one point

**Example #1**: Consider the graph of the function shown.

(a) Determine whether the instantaneous rate of change is positive or negative at each of the indicated points.

(b) Estimate the instantaneous rate of change at:
   (i) \( x = -3 \)  
   (ii) \( x = 2 \)

(c) What type of points exist at \( x = -3 \) and at \( x = 2 \)?
The instantaneous rate of change is **zero** at both the maximum and the minimum point.

- The slope of the tangent is horizontal here!

As a curve increases, the slopes of the tangents to the curve are **positive** on this interval.

As a curve decreases, the slopes of the tangents to the curve are **negative** on this interval.

- **At a maximum point**, the slopes of the tangents must change from positive to negative.

  ![Diagram showing a maximum point with positive and negative slopes]

- **At a minimum point**, the slopes of the tangents must change from negative to positive.

  ![Diagram showing a minimum point with negative and positive slopes]

**Example #2:**

For the function \( f(x) = x^3 - 27x + 1 \), verify that the point \((-3, 55)\) is either a maximum or a minimum.